

MAS 117

Introduction to Algebra

Assignment 5

For handing in on 14 February 2007

*Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the **RED** post-box on the **BASEMENT** of the Maths building before 16:00 on Wednesday.*

1 (20 marks) Find the greatest common divisor of 2262 and 3335, and express it in the form $2262u + 3335v$ for some integers u and v .

2 (20 marks) Find the greatest common divisor of the polynomials $x^3 + x^2 - 5x - 2$ and $x^4 + 4x^3 + 3x^2 - 2x - 1$, and express it in the form

$$(x^3 + x^2 - 5x - 2)u(x) + (x^4 + 4x^3 + 3x^2 - 2x - 1)v(x)$$

for some polynomials $u(x)$ and $v(x)$.

3 (40 marks) Here is a *definition* of the **factorial function**, which maps the natural numbers to the natural numbers. (Despite the curious notation, this really is a function!)

- $0! = 1$.
- If $n > 0$, then $n! = n \times (n - 1)!$.

(a) Prove by induction that this really does define a function; that is, prove that, for any natural number n , the rule given enables us to calculate $n!$.

(b) Prove by induction that, for $n \geq 1$,

$$2^{n-1} \leq n! \leq n^n.$$

4 (20 marks) Suppose that an operation $\circ$ on $\{p, q\}$ is given by the rule that

$$p \circ p = p \circ q = q \circ p = q \circ q = p.$$

Does this operation satisfy the associative law? The commutative law? The identity law? The inverse law?

5 Prize question A cash prize is offered for the best solution received to this question. Do not hand this in with the rest of your coursework; give your solutions to me.

Prove that

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) + \mathbf{v} \times (\mathbf{w} \times \mathbf{u}) + \mathbf{w} \times (\mathbf{u} \times \mathbf{v}) = \mathbf{0}$$

for any three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$. [I will prefer a proof which does not simply do a long and boring calculation in coordinates!]